INJECTIVE CHROMATIC NUMBER OF SOME GRAPH FAMILIES

ESTRADA, CELEDONIO IV L.,[1,*] ACILO, BARBIE B.,[1] MARTINEZ, JAMAICA STEPHANIE,[1] and SIMON, JOHN ALFRED C.[1]

estrada.c.bsmath@gmail.com[1] acilo.b.bsmath@gmail.com[1] martinez.js.bsmath@gmail.com[1] and simon.ja.bsmath@gmail.com[1]

[1]Bachelor of Science in Mathematics, College of Arts and Sciences, Eulogio "Amang" Rodriguez Institute of Science and Technology

Abstract

An injective coloring of a graph G is a vertex coloring where any two distinct vertices with a common neighbor receive distinct colors. Unlike proper coloring, injective coloring does not require adjacent vertices to have different colors unless they share a common neighbor. Although adjacent vertices are not automatically required to receive different colors, nonadjacent vertices may also be required to differ when they share a common neighbor. This study aimed to define and analyze injective coloring, establish its basic properties, and determine the exact injective chromatic numbers of selected graph families. The study employed descriptive and expository mathematical methods using definitions, examples, mathematical arguments, and proof techniques. Lower bounds were obtained by identifying vertices that must receive distinct colors, while upper bounds were established by constructing valid injective colorings. The two-step graph was also used to analyze common-neighbor relationships. The study established the general bounds involving the maximum degree and the relationship between the injective chromatic number and the distance-two chromatic number. Exact values were obtained for the selected families: ladder graphs have injective chromatic number three for order at least three; tadpole graphs have injective chromatic number three; helm graphs have injective chromatic number $n + 1$; fan graphs have injective chromatic number $n + 1$; friendship graphs have injective chromatic number $2n + 1$; and nontrivial generalized m-ary trees have injective chromatic number equal to their maximum degree, which is at most $m + 1$. The findings show that common-neighbor relationships, maximum degree, and two-step graph structure influence the number of colors required. The study provides a foundation for investigating other graph families, graph operations, and possible applications of injective coloring.

Keywords: injective chromatic number; injective coloring; graph families; graph theory; maximum degree

*Corresponding author / Presenter