ON REGAL COLORING OF SOME FAMILIES OF GRAPHS

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Abstract

Regal coloring is an edge-coloring concept in which every pair of adjacent vertices must have distinct induced color sets. For a vertex v, its induced color set is obtained from the intersection of the color sets assigned to its incident edges, given by

$$c'(v) = \bigcap_{e \epsilon v} c(e).$$

This study aimed to investigate regal coloring and determine the regal indices of selected families of graphs. Specifically, the study focused on fan graphs, friendship graphs, m-ary tree graphs, and homogeneous caterpillar graphs. The study employed descriptive and expository mathematical methods using established definitions, properties, graph constructions, and mathematical proofs. The regal index of a graph was determined by establishing lower bounds and constructing valid regal edge colorings to obtain corresponding upper bounds. The study established that the fan graph F_n, for $n \geq 2$, has regal index $reg(F_n) = 2$, while the friendship graph $Fr(n)$, for $n \geq 1$, has regal index $reg(Fr(n)) = 2$. The constructions for these graph families showed that two colors are sufficient to satisfy the regal coloring condition, while one color is insufficient. The study also established properties and bounds related to regal coloring and demonstrated how recurring structural patterns can be used in constructing regal edge colorings. The findings contribute to the study of regal coloring and provide a foundation for further investigation of regal indices in other graph families.

Keywords: regal coloring, regal index, edge coloring, graph coloring, graph families