ON DIVISOR FUNCTION OF SOME INTEGER SEQUENCES

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Abstract

The divisor function, denoted by $\tau(n)$, is a fundamental arithmetic function in Number Theory that counts the positive divisors of a given positive integer and can be determined through its prime factorization. This study investigates the properties, patterns, and behavior of the divisor function on selected classes of integer sequences, particularly positive consecutive integers. Specifically, it examines square numbers, square-free numbers, pronic numbers, triangular numbers, semiprime numbers, sphenic numbers, Mersenne primes, Fermat primes, Sophie Germain primes, safe primes, and sequences involving consecutive positive integers. A descriptive and expository research method was employed using established definitions, mathematical properties, prime factorization, and direct calculations to analyze divisor counts. The study established that the value of $\tau(n)$ is closely related to the structure and prime factorization of an integer. Square numbers have an odd number of divisors, while square-free numbers have divisor counts determined by the number of distinct prime factors. Semiprime and sphenic numbers also exhibit specific divisor-count patterns based on their prime factors. For Mersenne primes, Fermat primes, Sophie Germain primes, and safe primes, the divisor function is equal to two. Furthermore, for two consecutive positive integers whose divisor-function sum is six, the possible ordered divisor-count pairs are (2,4) and (4,2), while for three consecutive positive integers whose divisor-function sum is ten, the possible patterns are (2,4,4), (2,6,2), (4,2,4), and (4,4,2). The findings demonstrate that the divisor function provides a useful framework for identifying patterns among integer sequences and may contribute to mathematical applications involving prime numbers, factorization, and cryptography.

Keywords: Divisor Function; Number Theory; Integer Sequences; Prime Factorization; Consecutive Integers