Equitable Coloring of Some Families of Graphs

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Abstract

Equitable coloring is a vertex coloring of a graph in which no two adjacent vertices receive the same color and any two color classes differ in size by at most one. Unlike proper coloring, which requires only that adjacent vertices receive distinct colors, equitable coloring additionally imposes a balanced distribution of vertices among the color classes. This study aimed to define and analyze equitable coloring, establish its basic properties, and determine the exact equitable chromatic numbers of selected graph families, namely the double fan graph, centipede graph, ladder graph, and crown graph. The study employed descriptive and expository mathematical methods using definitions, examples, mathematical arguments, and proof techniques. Two basic properties of equitable coloring were established: first, in any equitable k-coloring of a graph of order n, every color class contains either ⌊n/k⌋ or ⌈n/k⌉ vertices; and second, when k divides n, every color class has exactly the same size, namely n/k. Lower bounds were obtained by identifying vertices that must receive distinct colors, while upper bounds were established by constructing valid equitable colorings. The main result of this study is the equitable chromatic number of the double fan graph, whose structure of two fan graphs sharing a common path makes the balanced distribution of its vertices among color classes the deciding factor in the number of colors required. The findings show that both the adjacency of vertices and the balanced distribution of vertices among the color classes influence the number of colors required, and that the structure and arrangement of the vertices of each graph determine its equitable chromatic number. The study provides a foundation for investigating other graph families and graph operations.

Keywords: Keywords: equitable coloring; equitable chromatic number; double fan graph; color classes; graph families