ON GAUSSIAN BINOMIAL COEFFICIENT

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Abstract

Gaussian binomial coefficients, also known as q-binomial coefficients, represent a significant generalization of ordinary binomial coefficients through the introduction of a weight parameter q. Despite their mathematical importance in combinatorics, algebra, and number theory, comprehensive and accessible explanations of these coefficients remain scarce in current literature. This study provided a rigorous theoretical examination of the definition, fundamental properties, and combinatorial applications of the Gaussian binomial coefficient. The study also explores the relationships between Gaussian binomial coefficients and other combinatorial constructs, including ordinary binomial coefficients, multinomial coefficients, and Fibonomial coefficients, demonstrating how the q-analogue unifies these concepts. Employing a theoretical and expository methodology grounded in logical analysis and exact definitions, the research systematically derived key properties including symmetry, boundary values, recurrence relations, and the limiting behavior as q approaches 1. The study demonstrated that the Gaussian binomial coefficient reduces to the classical binomial coefficient in the limit, establishing it as a one-parameter deformation of the ordinary coefficient. Furthermore, the finite-field subspace counting theorem was proven, showing that Gaussian binomial coefficients enumerate vector subspaces over finite fields. Applications in combinatorics and finite geometry were illustrated through concrete examples, including subspace counting and q-series identities. The exposition was structured to be self-contained and pedagogically linear, beginning with prerequisites and progressing through definitions to derived consequences. The findings revealed that Gaussian binomial coefficients serve as a bridge connecting classical combinatorics with advanced algebraic structures, with applications spanning number theory, finite geometry, and quantum algebra. This research contributed to mathematical discourse by providing a comprehensive pedagogical resource that makes these coefficients accessible to students, educators, and researchers while establishing a foundational reference for future investigations in quantum combinatorics and related disciplines.

Keywords: Gaussian binomial coefficient; q-analogue; combinatorics; quantum algebra; finite fields

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