On Royal Chromatic Indices of Some Families of Graphs

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Abstract

Royal coloring is a specialized concept in graph theory that extends conventional graph coloring by assigning nonempty subsets of a prescribed color set to the edges of a graph, subject to conditions on the colors induced at its vertices. This study, entitled *"On Royal Chromatic Indices of Some Families of Graphs,"* investigates the royal chromatic indices of selected families of graphs. Specifically, it aims to examine the fundamental concepts and properties of royal coloring, including strong royal coloring and its corresponding index, and to determine the royal chromatic indices of selected graph families. The study employs a descriptive and expository theoretical approach based on relevant definitions, assumptions, notations, lemmas, propositions, theorems, examples, computations, and mathematical proofs. Selected families of graphs, namely ladder, caterpillar, sunlet, fan, double fan, broom, double star, and friendship graphs, are examined through systematic constructions and mathematical arguments. The results establish the royal chromatic indices of the graph families considered and demonstrate how these values are related to their structural properties, particularly vertex degrees, adjacency relationships, and graph configurations. The study also provides the necessary theoretical background on strong royal coloring and its corresponding index to support the development of the concepts used in the investigation. Overall, the findings contribute to the understanding of royal coloring and its associated parameters and demonstrate the relevance of royal chromatic indices in studying subset-based edge colorings. The results may serve as a mathematical reference for students, educators, researchers, and future researchers interested in royal coloring, graph coloring, and related areas of graph theory. The study may also provide a foundation for further investigations of royal coloring and its variants in other families of graphs.

Keywords: Royal Coloring; Royal Chromatic Index; Strong Royal Coloring; Edge Coloring; Graph Theory