On Sum of all the Subsets of a Non-Empty Power Set Number

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Abstract

The Non-empty Subset Sum Number is obtained by adding the sums of all non-empty subsets of the first n positive integers. This study examines its mathematical structure and its connections to powers of two, triangular numbers, and Pascal's triangle. It aims to define the sequence, derive its closed-form expression, establish its properties, and determine its relationships with fundamental concepts in combinatorics and number theory. The study employs a theoretical and deductive approach using combinatorics, algebra, and number theory. The contribution of each element is determined by counting the number of subsets in which it occurs. Mathematical induction, recurrence relations, divisibility arguments, algebraic manipulation, and generating functions are used to derive and verify the results. The study establishes that the Non-empty Subset Sum Number can be expressed as the product of the sum of the first n positive integers and a power of two. This form demonstrates its relationship with triangular numbers and the binomial coefficients represented in Pascal's triangle. The study also establishes that the sequence is even for n greater than or equal to three, and exhibits exponential growth. The findings demonstrate that the Non-empty Subset Sum Number has a clear combinatorial structure connecting subset enumeration, powers of two, triangular numbers, and binomial coefficients. These results provide a basis for further studies on generalized subset-sum sequences and related combinatorial structures.

Keywords: Non-empty Subset Sum Number; Subset Sums; Powers of 2; Triangular Numbers; Pascal's Triangle